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RILL HYDRAULICS ON A SEMIARID HILLSLOPE, SOUTHERN ARIZONA

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ABSTRACT

Seventy field experiments were conducted in seven rills located on a semiarid rangeland hillslope underlain by gravelly soils at Walnut Gulch, Arizona. The rills, which are characterized by wide, shallow cross-sections and gravel-covered beds, have mean at-a-station hydraulic geometry exponents of $b = 0{\cdot}33, f = 0{\cdot}34$ and $m = 0{\cdot}33$. Although the differences between these values and typical values of $b = 0{\cdot}30, f = 0{\cdot}40$ and $m = 0{\cdot}30$ for cropland rills are not statistically significant, they are thought to be real, as cropland rills often have more rectangular cross-sections and steeper sides than the rangeland rills under study. For rills formed in silty loamy soils, Govers developed an empirical relation between mean flow velocity and discharge. Emphasizing the generality of this relation, he suggested that it may be used as a simple means of routing runoff through rills. He also noted that this relation appeared to be unaffected by either slope or soil materials. The present data represent rills underlain by coarser and somewhat more varied gravel-rich soils. These data do not conform to Govers' relation, and a multiple regression analysis reveals that slope and soil materials, either directly or indirectly through bed roughness, exert almost as much influence on flow velocity as does discharge. Three alternative methods are developed for predicting flow velocity in the rills under study. All three methods give good results with the largest root mean square deviation being $3{\cdot}115\,\mathrm{cm\,s^{-1}}$.

KEY WORDS rills; hillslopes; hydraulics; deserts; Arizona

INTRODUCTION

Rills are an integral part of the runoff system on many, if not most, arid and semiarid hillslopes. Overland flow collects into these microchannels and travels downslope as concentrated linear flow, which is deeper and faster than the more dispersed overland flow that occurs in interrill areas. It is therefore important that models of hillslope runoff explicitly distinguish rill flow from interrill flow in their routing schemes, particularly where such models are to be used to predict soil erosion (e.g. Nearing *et al.*, 1989). Before rill flow can be adequately modelled, however, empirical research is required on rill hydraulics to guide the modelling effort. Although there is a body of research on the hydraulics of rills on croplands (Meyer *et al.*, 1975; Lane and Foster, 1980; Foster *et al.*, 1984a,b; Line and Meyer, 1988; Gilley *et al.*, 1990; Govers, 1991, 1992), to our knowledge there has been no work on self-formed rills on rangelands. In this paper we report on a series of field experiments aimed at providing an initial data set and an analysis of rill hydraulics on semiarid rangeland hillslopes.

PREVIOUS WORK

Leopold and Maddock (1953) proposed that the hydraulic geometry of river channels could be characterized by a set of bivariate power relations relating channel width w, mean flow depth h, and mean flow velocity

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Table I. Hydraulic geometry exponents for rills formed in cropland soils

Relation	Exponent	Reference
Width vs. discharge	0·48	Lane and Foster (1980)
	Range: 0·14–0·47	Gilley *et al.* (1990)
	Mean: 0·30 ($N = 10$)	
Velocity vs. discharge	0·32	Meyer *et al.* (1975)
	0·32	Lane and Foster (1980)*
	0·28	Foster *et al.* (1984a)
	0·23, 0·26, 0·39	Line and Meyer (1988)
	0·29	Govers (1992)

* Reported by Moore and Foster (1990)

u to discharge Q:

$$w = aQ^b \tag{1}$$

$$h = cQ^f \tag{2}$$

$$u = kQ^m \tag{3}$$

where a, b, c, f, k, m are empirical constants such that $ack = 1$ and $b + f + m = 1$. Rill hydraulics may be characterized in a similar manner. In previous field and laboratory studies of rills developed in cropland soils, investigators have fitted equations in the form of Equations 1 and 3 to data collected at a cross-section. The values of the exponents obtained in these studies are summarized in Table I. Although Lane and Foster (1980) found that $b = 0{\cdot}48$, Gilley *et al.*'s (1990) extensive study of rills in 10 different soil types suggests that $b = 0{\cdot}30$ is more typical. The values of m reported in the literature also seem to cluster around 0·30. Accepting that $b = 0{\cdot}30$ and $m = 0{\cdot}30$, it follows that $f = 0{\cdot}40$. There are no published data on f for rills with which to compare this inferred value.

The tendency for the velocity exponent m to stay close to 0·30 is consistent with a recent analysis by Govers (1992) of 409 observations of rill velocity and discharge collected from the literature. Using non-linear regression, he obtained the equation

$$u = 6{\cdot}061Q^{0{\cdot}294} \tag{4}$$

where u is in cm s^{-1}, Q is in cm^3 s^{-1}, and the coefficient of determination $R^2 = 0{\cdot}73$. Govers claimed that the relation between u and Q appeared to be unaffected by slope or soil materials, though he noted that all his data came from rills formed in silty loamy soils. Govers attributed the lack of a relationship between velocity and slope to a tendency for bed roughness to increase as slope increases, thereby counterbalancing the tendency for velocity to increase with slope. He explained the absence of a relationship between velocity and soil materials in a similar manner: as materials become more resistant, rills tend to become narrower and flow velocity increases, on the one hand, while bed roughness increases and flow velocity decreases, on the other. These two opposing tendencies lead to a mean flow velocity that is more or less independent of soil materials. This conceptual model of Govers is summarized in Figure 1. Emphasizing the generality of Equation 4, Govers suggested that it may be used as a simple means of routing hillslope runoff through rills. All that is needed to employ Equation 4 in this manner is rill discharge, which can be computed from infiltration curves and a knowledge of flow pathways. Calculating u directly using Equation 4 therefore avoids the more conventional practice (e.g. Henderson and Wooding, 1964; Woolhiser *et al.*, 1970; Scoging, 1992) of computing u from a resistance equation, such as the Darcy–Weisbach or Chezy equation, and having to estimate the relevant resistance coefficient. If Govers is right about the generality of Equation 4, this equation provides a powerful modelling tool. Before it is used for modelling purposes, however, it should be tested against other data sets to verify its generality.

Against this background, field experiments were undertaken in seven rills developed on a single hillslope at Walnut Gulch Experimental Watershed, Tombstone, Arizona (31°43′N, 110°41′W). The aims of these

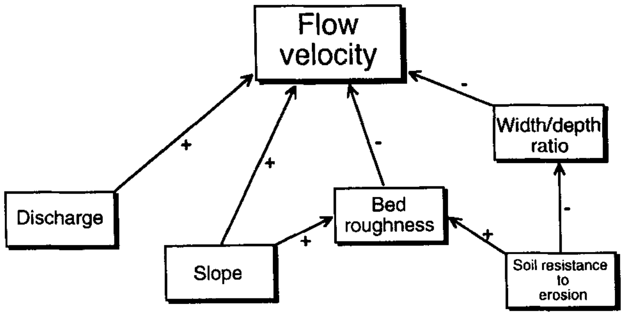

Figure 1. Flow diagram summarizing Govers' (1992) conceptual model of the factors influencing flow velocity in rills

experiments were (1) to characterize the hydraulics of self-formed rills on semiarid rangeland hillslopes, (2) to test whether Equation 4 applies to such rills, and (3) if it does not, to develop alternative equations for predicting flow velocity.

RILLS AND METHODS

The seven rills selected for study are located on a semiarid shrubland hillslope that is typical of the dissected piedmont in the lower part of the Walnut Gulch watershed. During the past century, open shrub communities have replaced grassland over much of the northern Sonoran and Chihuahuan Deserts (e.g. Glendening, 1952; Buffington and Herbel, 1965; Hastings and Turner, 1965; Gibbens and Beck, 1988), including the lower 60 per cent of the Walnut Gulch watershed (Renard, 1970). As the shrubs grew and the understorey grasses thinned, the A horizon of the pre-existing soil was selectively removed in the intershrub areas by a combination of rainsplash and overland flow, leaving behind a gravel lag. These gravel-covered surfaces have lower infiltration rates and offer lower resistance to flow than the former grass-covered surfaces (Abrahams *et al.*, 1994). Thus, not only does a given rainfall event generate more overland flow, but the flow is faster on the shrubland than on the former grassland (Parsons *et al.*, in press). In addition, mounds of fines beneath shrubs (Parsons *et al.*, 1992) create a microtopography that concentrates overland flow into gravel-covered intershrub areas, leading, on many hillslopes, to the formation of rills. These rills are broad, shallow, concave-upward troughs 0·8 to 1·0 m wide and 0·05 to 0·10 m deep. Their beds are usually covered with gravel, though on gentle footslopes the bed material may be largely sand. The rills vary in extent from one hillslope to another. On the hillslope under study they are up to 100 m long and reach to within 25 m of the divide.

The rills selected for study are all located downslope from the large (18 m × 35 m) interrill runoff plot whose hydraulics were described by Parsons *et al.* (1990). The hillslope is underlain by deep, calcareous, gravelly soils of the Hathaway series developed on well-cemented, coarse Quaternary alluvium (Gelderman, 1970). The particular reaches on which the experiments were performed are straight, or nearly so, and were selected to represent the range of rill morphologies displayed on the hillslope. Each reach is about 3 m long and contains a 0·5 m long test section located about 1·5 m from the upper end of the reach. Within each test section, three cross-sections were established 0·25 m apart. These cross-sections were measured prior to the experiments using a microrelief meter (McCool *et al.*, 1981) consisting of an array of 99 rods of the same length spaced 1 cm apart. The meter was levelled, the rods were lowered to the ground surface, and the positions of their upper ends were recorded on a strip chart.

To prevent scouring by the clear water used in the experiments, the surface of each rill was fixed by sprinkling on it a mixture of one part Elmer's wood glue and two parts water. This mixture infiltrated to a depth of several millimetres and sealed the ground surface without perceptibly altering its roughness. Underlying this methodology is the assumption that the hydraulics of clear-water flows over the fixed rill beds are similar to those of natural, sediment-laden flows over the same beds had they not been fixed. In the absence of field measurements of these rills during actual runoff events, there is no way of evaluating this assumption.

Table II. Rill characteristics

Rill	Slope (sine)	Microrelief (cm)	Gravel cover (%)	Mean gravel size (cm)
1	0·0375	1·968	70	2·069
2	0·0280	1·986	59	1·475
3	0·0130	1·160	74	1·114
4	0·0296	2·420	70	2·944
5	0·0559	2·508	77	3·451
6	0·0445	2·278	67	2·237
7	0·0133	0·292	29	0·569

However, it does not seem unreasonable, especially in view of the gravelly, and therefore resistant, nature of the rill beds.

The microrelief meter was also set up along the centreline of each rill, centred on the test section. The heights of the tops of the rods above the levelled base of the meter were recorded, and a linear regression performed between rod height and horizontal distance. The arc tangent of the regression slope coefficient is a measure of the local rill slope in degrees, and the sine of this angle S is taken as an estimate of the energy slope. The standard deviation of the residuals from the regression line M is employed as a measure of rill-bed roughness. Other measures of rill-bed roughness were obtained from the measurement of bed sediment size. Because many of the bed sediments were partly buried, the size of each particle was determined by measuring its upstream projected width. One hundred particles were sampled at 2 cm intervals along two lines 5 cm either side of the rill centreline. From these data the percentage of gravel-sized ($\geqslant 2$ mm) particles in the sample $\%G$ and the arithmetic mean size of the gravel D_G were calculated. The values of $S, M, \%G$, and D_G for the seven rills are listed in Table II.

Water simulating rill flow was introduced at the upper end of each reach in the following way. A trickle pipe was prepared by drilling evenly spaced holes along a 0·6 m long 2·5 inch polyvinyl chloride (PVC) pipe. One end of the pipe was blocked and a pressure gauge was attached close to this end. The other end of the pipe was connected to a gate valve, thence to a 7·5 kW pump, and finally to a water supply. The relation between discharge and water pressure was established, so for each experiment the inflow to the rill could be regulated simply by controlling the water pressure. The discharge from the trickle pipe was directed into a levelled trough from which it overflowed across a short canvas apron into the rill.

Ten experiments were conducted on each rill with inflow rates of 555, 787, 967, 1118, 1252, 1486, 1689, 1872, 2039 and 2194 $cm^3 s^{-1}$. The only exception was for rill 1, where the highest inflow rate was 2202 rather than 2194 $cm^3 s^{-1}$. The outflow rate for each rill during each experiment was determined by averaging the discharges calculated from two timed volumetric samples. Because the ground surface of each reach was sealed prior to the experiments, infiltration rates were negligible and the measured inflow and outflow rates were generally very similar. As inflow rates were deemed to be measured more reliably than outflow rates, where our measurements yielded an outflow rate greater than the inflow rate, the discharge Q at the centre of the test section was set equal to the inflow rate. On the other hand, where our measurements gave an outflow rate less than the inflow rate, the discharge at the test section was calculated by assuming that infiltration and evaporation rates were uniform along the length of the reach. Finally, flow width was measured at the three cross-sections and averaged to yield mean flow width w for the test section. The data for Q and w for each experiment are reported in Table III.

Knowing Q and w, to obtain the remaining basic hydraulic variables one must measure either mean flow velocity, mean flow depth or flow cross-sectional area, then one may compute the other two. In this set of experiments we chose to measure flow cross-sectional area. The method employed assumes that the water surface is always horizontal across the cross-section. During each experiment i, flow depth h_i was measured at a fixed point on each cross-section with a millimetre scale, and flow width w_i was measured with a centimetre scale. Using a drawing of the rill cross-section obtained from the microrelief meter, the area A_1 of the flow cross-section at the lowest discharge ($i = 1$) was measured with a polar planimeter after locating the

water surface from a knowledge of h_1 and w_1. The area A_2 was then obtained by adding the rectangle $w_2(h_2 - h_1)$ to A_1, and so on. The mean cross-sectional area for the test section A for the ith experiment was calculated by averaging the values of A_i for the three cross-sections. Likewise, the mean flow depth for the test section h for the ith experiment was computed by averaging the values of A_i/w_i for the three cross-sections. Finally, the mean flow velocity for the test section u for each experiment was determined by dividing Q by A, and the Darcy–Weisbach friction factor f was computed using

$$ff = 8ghS/u^2 \tag{5}$$

where g is the acceleration due to gravity. The data from A, h, u and ff for each experiment are given in Table III.

HYDRAULIC GEOMETRY

Linear regressions were performed for $\log w$ vs. $\log Q$, $\log h$ vs. $\log Q$ and $\log u$ vs. $\log Q$ for each rill to obtain hydraulic geometry relations in the form of Equations 1 to 3. The exponents of the derived relations and their associated correlation coefficients are reported in Table IV. Inasmuch as all but three of the 21 correlation coefficients exceed 0·90, the regression equations can be considered good estimators of their equivalent structural relations, and the computed exponents can be presumed to approximate their true values. This presumption is supported by the fact that $b + f + m$ in all cases is equal to or close to 1.

The mean values of b, f and m for the seven rills at Walnut Gulch are 0·33, 0·34 and 0·33, respectively (Table IV). Although these values are not significantly different (at the 0·05 level, which is used throughout this study) from the typical values of $b = 0{\cdot}30, f = 0{\cdot}40$ and $m = 0{\cdot}30$ for cropland rills (Table I), the differences may in fact be real and meaningful. The ratios b/f and m/f are both 0·75 for cropland rills and 0·97 for the Walnut Gulch rills. Inasmuch as at-a-station hydraulic geometry exponents are closely related to channel cross-sectional form (Knighton, 1975), the lower b/f and m/f ratios for the cropland rills can probably be attributed to these rills often having rectangular cross-sections with near-vertical sides, whereas the Walnut Gulch rills possess wide, shallow cross-sections with gently sloping sides. Such an interpretation is consistent with Leopold and Miller's (1956) finding that ephemeral river channels have higher b/f and m/f ratios than do 'average' rivers.

FACTORS INFLUENCING FLOW VELOCITY

Govers (1992, p. 515) proposed that Equation 4 represents a general relation between u and Q for rills, and he claimed that 'there is no important influence of slope and/or soil material characteristics on flow velocities'. Govers noted, however, that the rills he analysed had slopes ranging from 0·035 to 0·141 and were formed exclusively in silty loamy soils. By contrast, our rills at Walnut Gulch have gentler gradients ($0{\cdot}0133 \leqslant S \leqslant 0{\cdot}0559$: Table II) and are underlain by coarser and somewhat more varied gravel-rich soils. Thus, these rills afford a useful opportunity to test the generality of Govers' relation and the validity of his conclusion that slope and/or soil materials have little effect on rill velocity. As a first step, we plotted u against Q for the Walnut Gulch rills and on the same graph reproduced Govers' data and Equation 4 (Figure 2). It is immediately apparent that the Walnut Gulch data plot below Govers' data and Equation 4, signifying that at a given discharge, flow velocities are smaller in the rills at Walnut Gulch than in those analysed by Govers. Logically, the smaller velocities are due to the gentler slopes and/or the coarser soil materials at Walnut Gulch. Thus not only is the generality of Equation 4 impugned, but Govers' claim that slope and soil materials have a negligible effect on flow velocity is called into question. The effect of slope and soil materials on flow velocity is examined more closely in the following analysis.

To test Govers' conceptual model of the controls of flow velocity (Figure 1), it is necessary to obtain a meaningful measure of soil resistance to erosion. Given that the soil materials at Walnut Gulch consist of rounded gravels of various sizes supported in a matrix of fines, the erosion of which gives rise to a gravel lag, such a measure is difficult to obtain and so no attempt was made to do so. Instead, Govers' model was simplified, as shown in Figure 3, and the simplified model tested in the following analysis. If Govers'

Table III. Hydraulic data

Rill number	Q ($cm^3 s^{-1}$)	w (cm)	A (cm^2)	h (cm)	u ($cm s^{-1}$)	ff	h/D	h/D_{84}
1	541	36·8	39·4	1·08	13·8	1·67	0·73	0·36
	756	40·5	47·5	1·18	15·9	1·37	0·80	0·39
	945	42·5	56·0	1·33	16·9	1·37	0·90	0·44
	1090	46·3	60·6	1·33	18·0	1·21	0·90	0·44
	1245	46·0	63·9	1·40	19·5	1·08	0·95	0·47
	1453	47·8	69·3	1·48	21·0	0·99	1·01	0·49
	1689	53·0	77·0	1·46	21·9	0·89	0·99	0·49
	1851	54·3	82·5	1·53	22·4	0·90	1·04	0·51
	2028	55·8	91·1	1·66	22·3	0·99	1·12	0·55
	2202	60·3	96·8	1·61	22·7	0·92	1·09	0·54
2	531	37·8	31·4	0·84	16·9	0·64	0·81	0·49
	734	40·7	41·1	0·94	17·9	0·65	0·91	0·55
	922	46·0	46·6	0·95	19·8	0·53	0·91	0·56
	1048	49·2	53·2	1·01	19·7	0·57	0·98	0·60
	1183	50·7	61·6	1·22	19·2	0·73	1·18	0·72
	1409	54·8	69·0	1·26	20·4	0·66	1·21	0·74
	1588	57·0	74·7	1·31	21·3	0·64	1·26	0·77
	1760	58·3	84·4	1·45	20·9	0·73	1·40	0·85
	1924	59·2	89·3	1·51	21·5	0·71	1·46	0·89
	2195	58·3	92·2	1·58	23·8	0·61	1·53	0·93
3	489	44·3	35·1	0·79	14·0	0·41	0·93	0·47
	735	44·0	52·8	1·18	13·9	0·62	1·39	0·70
	919	45·8	57·3	1·24	16·0	0·49	1·46	0·73
	1056	47·7	62·5	1·31	16·9	0·47	1·54	0·77
	1187	47·8	65·8	1·37	18·0	0·43	1·61	0·80
	1486	50·0	69·2	1·38	21·5	0·30	1·62	0·81
	1689	51·7	78·7	1·52	21·5	0·34	1·79	0·89
	1845	55·7	83·5	1·52	22·1	0·32	1·79	0·89
	2039	59·2	88·3	1·50	23·1	0·29	1·77	0·88
	2195	61·3	92·2	1·51	23·8	0·27	1·78	0·89
4	554	25·3	42·8	1·70	12·9	2·35	0·83	0·39
	749	26·7	50·1	1·87	15·0	1·94	0·91	0·43
	967	27·7	52·9	1·91	18·3	1·33	0·93	0·43
	1114	28·7	56·9	1·97	19·6	1·20	0·97	0·45
	1206	28·8	62·7	2·16	19·2	1·36	1·06	0·49
	1486	30·2	68·7	2·27	21·6	1·13	1·11	0·52
	1666	30·7	70·5	2·30	23·6	0·95	1·12	0·52
	1773	31·3	80·1	2·55	22·1	1·21	1·25	0·58
	1915	32·3	84·9	2·62	22·6	1·19	1·28	0·60
	2156	33·5	87·2	2·61	24·7	0·99	1·28	0·59
5	509	21·5	32·9	1·55	15·5	2·84	0·57	0·26
	737	23·5	42·4	1·87	17·4	2·71	0·69	0·31
	890	28·0	45·2	1·63	19·7	1·84	0·60	0·27
	1069	30·3	54·5	1·81	19·6	2·06	0·66	0·30
	1185	30·0	57·5	1·92	20·6	1·98	0·70	0·32
	1397	34·5	66·1	1·92	21·1	1·88	0·70	0·32
	1634	35·3	69·6	1·98	23·5	1·57	0·73	0·33
	1803	36·5	73·2	2·01	24·6	1·45	0·74	0·33
	1994	37·8	77·0	2·04	25·9	1·34	0·75	0·34
	2108	38·5	86·7	2·25	24·3	1·67	0·83	0·38
6	521	30·5	30·3	0·99	17·2	1·17	0·65	0·30
	700	32·3	33·6	1·04	20·9	0·83	0·68	0·31
	948	33·5	42·0	1·25	22·6	0·86	0·82	0·38
	1048	35·8	52·8	1·47	19·8	1·30	0·96	0·44

Table III Continued

Rill number	Q ($cm^3 s^{-1}$)	w (cm)	A (cm^2)	h (cm)	u ($cm\,s^{-1}$)	ff	h/D	h/D_{84}
6 (cont.)	1184	37·5	58·5	1·55	20·2	1·32	1·01	0·47
	1486	43·5	67·2	1·53	22·1	1·10	1·00	0·46
	1649	46·8	70·7	1·50	23·3	0·96	0·98	0·46
	1852	54·5	74·7	1·38	24·8	0·78	0·90	0·42
	2039	57·8	80·5	1·39	25·3	0·76	0·91	0·42
	2166	61·5	86·7	1·41	25·0	0·79	0·92	0·43
7	527	29·7	24·1	0·81	21·9	0·18	3·44	2·03
	760	31·3	31·2	1·00	24·4	0·18	4·24	2·50
	947	32·8	33·4	1·02	28·3	0·13	4·32	2·55
	1084	34·7	38·3	1·10	28·4	0·14	4·67	2·75
	1252	35·8	41·1	1·15	30·5	0·13	4·85	2·86
	1486	37·0	48·7	1·31	30·5	0·15	5·55	3·27
	1689	38·2	51·9	1·35	32·6	0·13	5·73	3·38
	1872	38·7	56·3	1·45	33·3	0·14	6·15	3·63
	2039	41·2	61·0	1·48	33·4	0·14	6·27	3·70
	2195	41·3	67·8	1·64	32·4	0·16	6·94	4·09

model is correct, the interactions between slope, bed roughness and width/depth ratio should be such that the analysis will show that these variables have a negligible effect on flow velocity.

The analysis falls into two parts. First, a step-wise multiple regression was undertaken to identify the variables that affect velocity. In this regression, log u was the dependent variable and $\log Q$, $\log S$, $\log M$, $\%G$, $\log D_G$ and $\log w/h$ were the independent variables, with $\log M$, $\%G$ and $\log D_G$ being different measures of bed roughness. At each step, the independent variable that increased the explained variance by the largest amount entered the regression. Entry of variables ceased when the increment to the explained variance was no longer significantly different from zero. Log Q entered the regression first, followed by $\%G$, $\log S$ and $\log M$, bringing the explained variance to 90·1 per cent (Equation 6: Table V). Thus variables representing discharge, slope and bed roughness appear in the regression equation, but width/depth ratio does not. Width/depth ratio was therefore considered to have no effect on flow velocity and was excluded from the second part of the analysis.

In the second part, the relative importance of the factors influencing flow velocity was evaluated by calculating the part coefficients of determination associated with the variables representing these factors. The part coefficient of determination associated with a particular independent variable or set of independent variables is the absolute increase in explained variance due to the entry of that variable or set of variables into the regression after the other independent variables have already entered. Thus the part coefficient of determination between log u and log Q is equal to the difference between the R^2 values for Equations 6 and 7

Table IV. Hydraulic geometry exponents and correlation coefficients

Rill	Width exponent, b	Correlation coefficient	Depth exponent, f	Correlation coefficient	Velocity exponent, m	Correlation coefficient	$b+f+m$
1	0·34	0·98	0·29	0·98	0·37	0·99	1·00
2	0·35	0·98	0·48	0·98	0·20	0·95	1·03
3	0·22	0·91	0·38	0·92	0·41	0·97	1·01
4	0·20	0·99	0·34	0·97	0·46	0·98	1·00
5	0·43	0·99	0·21	0·89	0·35	0·98	0·99
6	0·51	0·95	0·26	0·77	0·23	0·89	1·00
7	0·24	0·99	0·46	0·99	0·29	0·97	0·99
Mean	0·33		0·34		0·33		1·00

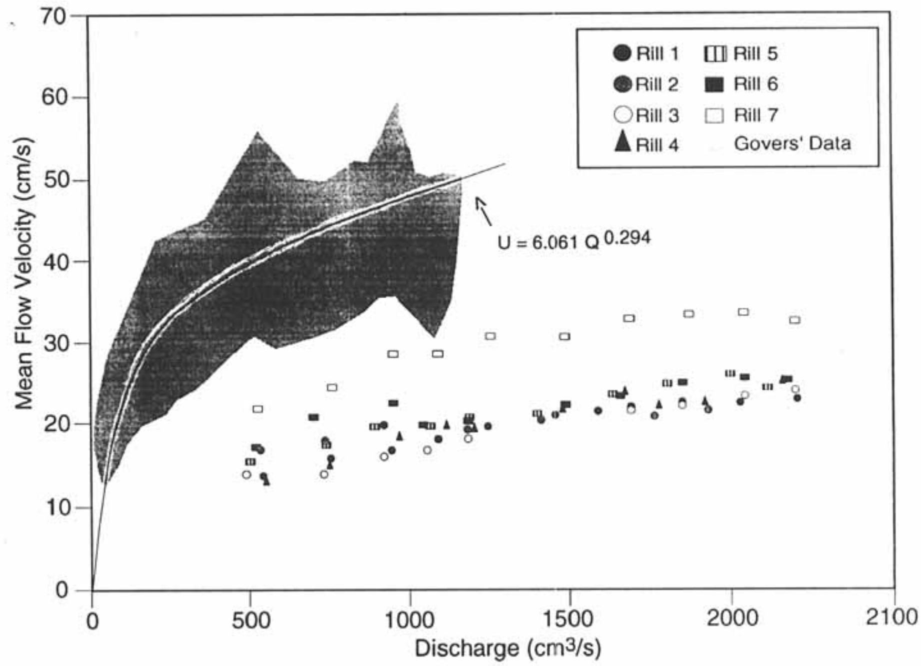

Figure 2. Graph of mean flow velocity against discharge showing the data from the present experiments conducted on seven rills together with Govers' (1992) data (shaded) and best-fit equation

(Table V); that between $\log u$ and $\log S$ is equal to the difference between the R^2 values for Equations 6 and 8; that between $\log u$ and the two variables representing bed roughness, $\log M$ and $\%G$, is equal to the difference between the R^2 values for Equations 6 and 9; and that between $\log u$ and the combined slope and bed roughness variables is equal to the difference between the R^2 values for Equations 6 and 10. These part coefficients of determination show: (1) that discharge has the greatest influence on flow velocity, accounting for 47·3 per cent of the variation in velocity independently of slope and bed roughness; (2) that bed roughness is second in importance, accounting for 36·5 per cent of the variation in velocity independently of discharge and slope; (3) that slope is of minor significance, accounting for only 6·0 per cent of the variation in velocity independently of discharge and bed roughness; and (4) that slope and bed roughness together exert almost as much influence as discharge on flow velocity, accounting for 41·4 per cent of the variation in velocity independently of discharge. Given this final point, and recalling that bed roughness is controlled by slope and soil materials (Figure 1), it follows that slope and soil materials, either directly or indirectly through bed roughness, have almost as large an effect as discharge on flow velocity. This finding contradicts Govers' conclusion that slope and/or soil materials have little effect on flow velocity and explains why the Walnut Gulch data plot below Equation 4 in Figure 2.

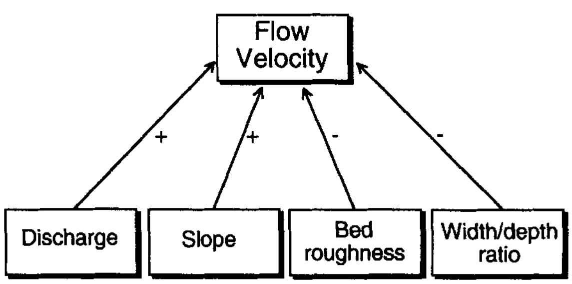

Figure 3. Flow diagram portraying a simplified version of Govers' (1992) conceptual model in which soil resistance to erosion is omitted

Table V. Multiple regressions

Equation number	Regression equation	Coefficient of determination	Part coefficient of determination	
			Variable(s)	Coefficient
(6)	$\log u = 0.733 + 0.329 \log Q - 0.00178\,\%G + 0.180 \log S - 0.188 \log M$	0.901		
(7)	$\log u = 1.754 - 0.00178\,\%G - 0.192 \log M + 0.180 \log S$	0.428	$\log Q$	0.473
(8)	$\log u = 0.517 + 0.329 \log Q - 0.00326\,\%G - 0.0177 \log M$	0.841	$\log S$	0.060
(9)	$\log u = 0.161 + 0.333 \log Q - 0.0873 \log S$	0.536	$\%G, \log M$	0.365
(10)	$\log u = 0.292 + 0.334 \log Q$	0.487	$\log S, \%G, \log M$	0.414

PREDICTING FLOW VELOCITY

The foregoing analysis indicates that while it might be possible to use Equation 4 to predict flow velocity through rills in silty loams, a simple bivariate relation between u and Q will not suffice for rills formed in a wider range of soil materials, such as those at Walnut Gulch. Instead, a more complex multivariate relation is called for that takes into account, in addition to discharge, slope and bed roughness. Equation 6 might conceivably serve this purpose, but it has the disadvantage of including among its predictor variables M, the measurement of which requires a microrelief meter. Inasmuch as a suitable microrelief meter may not always be available, this requirement limits the utility of Equation 6. In what follows, therefore, three alternative methods of predicting u are developed and compared. These methods are each more practical than Equation 6 in that they employ gravel size or gravel cover, which are more easily measured than M, to represent bed roughness.

Method 1

In an effort to derive a multivariate equation that might be used to predict u directly, a stepwise regression was performed between $\log u$ as the dependent variable and $\log Q$, $\log S$, $\%G$ and $\log D_G$ as the independent variables, that is, the independent variables representing discharge, slope and bed roughness without $\log M$. Log Q, $\%G$ and $\log S$ entered the regression, yielding the equation

$$\log u = 0.672 + 0.330 \log Q - 0.00415\,\%G + 0.0664 \log S \tag{11}$$

with $R^2 = 0.859$. A comparison of the R^2 values for Equations 6 and 11 reveals that the omission of $\log M$ reduces the explained variance by only 4·2 per cent. Thus the cost of excluding $\log M$ in terms of a reduced ability to predict u appears minimal, and in most circumstances will be outweighed by the practical advantage of not requiring a microrelief meter.

Method 2

Govers' (1992) suggestion that rill flow might be modelled by developing an equation to predict flow velocity directly was a departure from convention. The more traditional approach has been to estimate *ff* (or some other resistance coefficient) and then calculate u from Equation 5 (or some other resistance equation). The viability of this approach, however, depends on the ability to estimate *ff* both easily and reliably. Two methods will now be explored for estimating *ff* and then calculating u.

The first method of estimating *ff* is suggested by a study of interrill overland flow on the same hillslope at Walnut Gulch on which the rills are located. From 73 trickle experiments on gravel-covered interrill runoff plots, Abrahams *et al.* (1994) obtained the equation

$$\log ff = -1.099 + 0.024\,\%G - 0.313 \log Re + 0.915 \log D_G \tag{12}$$

where Re is the Reynolds number and $R^2 = 0.639$. They noted that if Re is omitted from the equation, R^2

diminishes by only 0·076, indicating that *ff* can be predicted almost as well without *Re* as with it. This finding implies that *ff* for interrill overland flow on these gravel-covered surfaces is controlled very largely by surface roughness, and that flow rate is of relatively little importance. This is not surprising given that resistance to interrill overland flow consists almost entirely of form and/or wave resistance induced by surface roughness (Abrahams *et al.*, 1992). Inasmuch as resistance to flow in rills is also dominated by form and/or wave resistance (Foster *et al.*, 1984b), *ff* for rill flow might also be expected to be controlled very largely by surface roughness.

To investigate whether *ff* for rill flow at Walnut Gulch can be well predicted by an expression similar to Equation 12 containing measures of surface roughness and flow rate, a stepwise multiple regression was performed with $\log ff$ as the dependent variable and $\%G$, $\log D_G$ and $\log Q$ as the independent variables. Log D_G was the first variable to enter the regression, accounting for 92·2 per cent of the variance in $\log ff$. Log Q then entered, increasing the explained variance to 95·1 per cent. The final regression equation is

$$\log ff = 0.512 + 1.383 \log D_G - 0.317 \log Q \tag{13}$$

The inclusion of $\log Q$ among the predictor variables improves the explained variance by only 2·9 per cent. Thus, as in the case of interrill overland flow, in rill flow *ff* is controlled very largely by surface roughness, and flow rate is of little importance.

In order to calculate u using Equation 5 and a knowledge of *ff*, one also needs to know h and S. The latter variable is a surface property that is independent of flow rate and easily measured. On the other hand, h is dependent on flow rate and is difficult to measure. However, a multiple regression analysis reveals that h, like *ff*, can be estimated from D_G and Q. The appropriate regression equation is

$$\log h = -0{\cdot}981 + 0{\cdot}286 \log D_G + 0{\cdot}349 \log Q \tag{14}$$

with $R^2 = 0{\cdot}666$. Thus, a second method of predicting u is to use Equation 5 in combination with Equations 13 and 14.

Method 3

A third method of predicting u has its origins in the study of rivers with large-scale bed roughness. Bed roughness is described as large-scale where the bed material protrudes through the flow, a condition requiring that the ratio of mean flow depth h to mean sediment size D be less than about 4 (Bathurst, 1982). During the rill experiments at Walnut Gulch, h/D was always less than 4, except during the experiments in rill 7, where h/D ranged up to 6·94 (Table III). Thus, to all intents and purposes, bed roughness during the rill experiments may be regarded as large-scale.

From their studies of rivers with large-scale roughness in Utah, Judd and Peterson (1969) concluded that flow resistance was a power function of channel cross-sectional shape, relative submergence and roughness concentration. Reasoning that channel cross-sectional shape is a function of relative submergence, Bathurst (1982) argued that it should be possible to develop an even simpler expression for flow resistance, in which resistance is an empirical function of relative submergence and roughness concentration. Guided by this line of thinking, and using h/D_{84} as a measure of relative submergence (Table III) and $\%G$ as a measure of roughness concentration, a stepwise multiple regression was performed with $\log ff$ as the dependent variable and $\log h/D_{84}$ and $\%G$ as the independent variables. Only the former independent variable entered the regression, yielding the equation

$$\log ff = -0{\cdot}350 - 1{\cdot}079 \log (h/D_{84}) \tag{15}$$

with $R^2 = 0{\cdot}850$.

In order to utilize Equation 15 to estimate *ff*, one needs data on both h and D_{84}. The latter variable can be readily obtained from measurements of the size of gravel on the rill bed. The former variable, on the other hand, varies with discharge and is difficult to measure. However, it may be estimated using Equation 14, which requires a knowledge of Q and D_G. Thus, a third method of estimating *ff* is to use Equation 5 in combination with Equations 14 and 15.

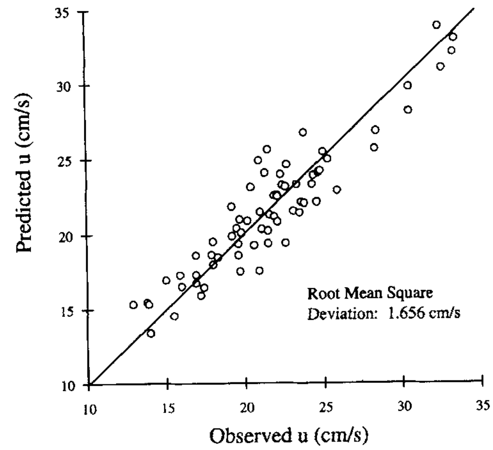

Figure 4. Graph of predicted mean flow velocity against observed mean flow velocity where predicted velocity was obtained by method 1

Comparison of methods

Figures 4, 5 and 6 contain plots for the 70 rill experiments of observed flow velocity against flow velocity predicted by each of the three methods. In all three figures the data plot fairly symmetrically around the line of perfect agreement, indicating qualitatively that all three methods give unbiased estimates of u. A comparison of the root mean square deviations for the three methods reveals that the first method is a better predictor of u than the second which, in turn, is better than the third. However, considering that the largest root mean square deviation is only $3{\cdot}115\,\mathrm{cm\,s^{-1}}$, all three methods may be viewed as providing satisfactory predictions of u.

CONCLUSIONS

The main conclusions of this paper are as follows.

(1) Rills developed on a semiarid rangeland hillslope underlain by gravelly soils have wide, shallow cross-sections and gravel-covered beds. The mean at-a-station hydraulic geometry exponents for these rills are $b = 0{\cdot}33, f = 0{\cdot}34$ and $m = 0{\cdot}33$. Although the differences between these values and typical values of $b = 0{\cdot}30, f = 0{\cdot}40$ and $m = 0{\cdot}30$ for cropland rills are not statistically significant, they are thought to be

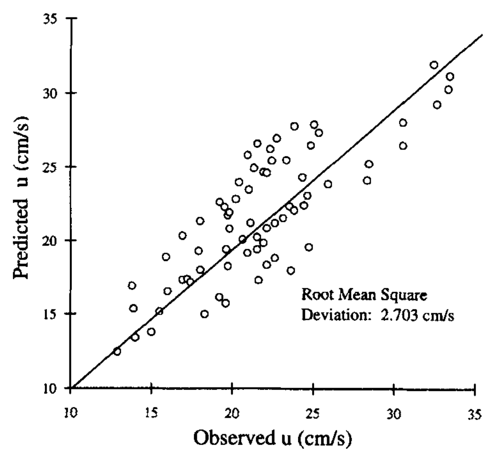

Figure 5. Graph of predicted mean flow velocity against observed mean flow velocity where predicted velocity was obtained by method 2

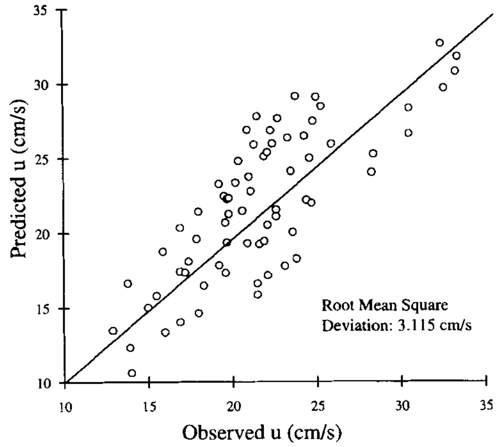

Figure 6. Graph of predicted mean flow velocity against observed mean flow velocity where predicted velocity was obtained by method 3

real, as cropland rills often have more rectangular cross-sections and steeper sides than the rangeland rills under study.

(2) For rills formed in silty loamy soils, Govers (1992) developed an empirical relation between mean flow velocity and discharge. Emphasizing the generality of this relation, he claimed that it appeared to be unaffected by slope or soil materials. The present data represent rills underlain by coarser and somewhat more varied gravel-rich soils. These data plot well below Govers' relation, and a multiple regression analysis indicates that, in addition to discharge, flow velocity is related to bed roughness and slope. Because bed roughness is a function of slope and soil materials, it is evident that, contrary to Govers' claim, slope and soil materials do have a significant influence on flow velocity. Indeed, an analysis of part coefficients of determination reveals that together, slope and soil materials are almost as important as discharge as controls of flow velocity.

(3) Using multiple regression analysis, three alternative methods are developed for predicting flow velocity in the rills under study. The first method predicts velocity directly from measurements of discharge, gravel cover and slope. The second and third methods involve measuring slope, estimating friction factor and mean flow depth from discharge and gravel size, and computing flow velocity using the Darcy–Weisbach resistance equation. A comparison of observed and predicted velocities for the three methods reveals that while the first method is better than the second which, in turn, is better than the third, all three methods provide quite good predictions of flow velocity.

(4) Additional studies of self-formed rills in other rangeland hillslopes are needed to establish the range of conditions over which these three methods of predicting flow velocity apply. As additional data from cropland as well as rangeland rills become available, it may be possible to develop a general equation for predicting flow velocity in all rills. Because resistance to flow in rills is dominated by form and/or wave resistance, such an equation might be expected to include measures of both bed roughness and flow rate among the predictor variables.

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